BMEN 343 **NAME:**____________________________________

ID #:______________________________________

HOMEWORK #3 (85 PTS)

DUE: 16 October 2009 at the BEGINNING OF CLASS:
There are 2 pages (front and back) to this homework. Answers should be given neatly, in order, and in the space provided or in stapled attached pages if necessary. Show work for full credit. Put the final answer in a box when appropriate.

True or False:

1. **(2 pt)** **T** F In order to form a substitutional solid solution with complete solubility, the elements should have the same valency.
True.

2. **(2 pt)** **T** F The driving force for steady-state diffusion is the concentration gradient.
True:

3. **(2 pts)** **T** F Heat treatment of an aluminum-silver diffusion couple (i.e. an Al bar in contact with a Ag bar) can lead to interdiffusion.
True:

4. **(2 pts)** T **F** For steady-state diffusion, diffusion flux is dependent on time.
False:

Brief Answer

4. Name the principle elements of the following alloys (i.e. those present in highest amounts). Provide the answer in terms of the atomic symbol **and** "name" of each element (e.g. "Ag/silver"). (Note: not necessary to list %'s of each element in alloy)

a. Pewter (2 major components) **(2 pts)**: Sn/tin (93%), Sb/antimony (6%) *[and also Cu/copper (1%)]*

(in some "older" forms of pewter, Cu was the 2nd highest in concentration).

b. Brass (2 major components) **(2 pts)**: Cu/copper and Zn/zinc

c. Stainless steel 316L (3 major components) **(3 pts)**: Fe/iron (60-65%), Cr/chromium (17-19%), and Ni/nickel (12-14%)

d. Haynes-Stellite 25 or ASTM-F90: (4 major components) **(4 pts)**: Co/cobalt (45.5-56.2%), Cr/chromium (19.0-21.0%), W/tungsten (14-16%); Ni/nickel (9-11%)

e. 55-Nitinol (2 major components) **(2 pts)**: Ni (50-55%), Ti (~50-45%)

Problem Solving

Calculations 5-9: *See the back of this page*. Answers should be completed on *separate pages* and ***stapled*** to this sheet. Show all work for full credit. Provide answers in a neat and orderly fashion. **Place a box around final answers.**

5. **(14 pts) (a)** For titanium (Ti), calculate the fraction of vacancies at **(i)** room temperature (25 °C) and at **(ii)** 950 °C. Assume the energy of vacancy formation is 1.2 eV/atom **(8 pts)**. **(b)** Based on the calculation, **one in how many sites** are vacant at **(i)** 25 °C and at (ii) 950 °C **(6 pts)**.

a.

(i) **(4 pts)** at RT

$$\frac{N_v}{N} = \exp\left(-\frac{Q_v}{kT}\right) = \exp\left[-\frac{(1.2ev/atom)}{(8.62x10^{-5}eV/atomK)(298K)}\right] = \boxed{5.15x10^{-21}}$$

(ii) **(4 pts)** at 950 C

$$\frac{N_v}{N} = \exp\left(-\frac{Q_v}{kT}\right) = \exp\left[-\frac{(1.2ev/atom)}{(8.62x10^{-5}eV/atomK)(1223K)}\right] = \boxed{1.14x10^{-5}}$$

b.

(ii) **(3 pts)** at RT

[1 / 5.15 x 10^{-21}] -- Therefore, 1 in 10^{20} (or 1 in 1.9 x 10^{20}) sites are vacant.

(ii) **(3 pts)** at 950 C

[1 / 1.14 x 10^{-5}] -- Therefore, 1 in 87,719 sites are vacant.

6. **(14 pts) (i)** Would you predict that more Pd (palladium) or more Co (cobalt) could be dissolved in Al (aluminum)? Consult the table below. *Explain your answer based on all 4 Hume-Rothery Rules for full credit* (**8 pts**). **(ii)** For Al-Pd and Al-Co, identify what they would form: (a) a substitutional solid of complete solubility, (b) a substitutional solid of incomplete solubility, or (c) an interstitial solid solution. **(6 pts)**

Element	*Atomic Radius (nm)*	*Crystal Structure*	*Electro-nega-tivity*	*Valence*
Cu	0.1278	FCC	1.9	+2
C	0.071			
H	0.046			
O	0.060			
Ag	0.1445	FCC	1.9	+1
Al	0.1431	FCC	1.5	+3
Co	0.1253	HCP	1.8	+2
Cr	0.1249	BCC	1.6	+3
Fe	0.1241	BCC	1.8	+2
Ni	0.1246	FCC	1.8	+2
Pd	0.1376	FCC	2.2	+2
Zn	0.1332	HCP	1.6	+2

Element	Radius (nm)	Crystal Structure	Electronegativity	Valency
Al (hoste or "A")	0.1431	FCC	1.5	+3
Pd (solute or "B")	0.1376 **(-4%)**	FCC (same)	2.2 (somewhat different)*	+2 (lower)
Co (solute or "B")	0.1253 **(-14%)**	HCP (different)	1.8 (similar)	+2 (lower)

Part i:

(1) **Atomic size factor or 15% rule**: atomic size of A (host) and B (solute) within 15% of each other

Although both within 15% the size of Al, Pd is closer in size to Al than Co **(2 pts)**

(2) **Crystal Structure Rules:**

Pd and Al have the same crystal structure but Co's is different. **(2 pts)**

(3) **Valency Rule:** valency of elements should be same for "complete" but B>A is better than B<A

Pd and Co's valency are both +2 which is "similar" to Al's valency of +3 (NEED EXACTLY SAME FOR COMPLETE SOLUBILITY) **(2 pts)**

(4) **Electronegativity Rule:** elements should have similar electronegativities

Pd's electronegativity is not as similar to Al's as is Co's.**(2 pts)**

Because of the closer size and same crystal structure, more Pd should dissolve in Al.

Part ii:

Al-Pd: (b) **(3 pts)** **(Note: since valencies are not exactly the same and electronegativities are > ~±0.4)**
Al-Co: (b) **(3 pts)**

7. **(12 pts)** What is the composition, in atom percent, of an alloy that consists of 100 lb_m Ti, 20 lb_m Al, and 5 lb_m V?

$lb_m \rightarrow g$

$$m'_{Ti} = (100\ lb_m)(453.6\ g/lb_m) = 45{,}360\ g\ Ti$$

$$m'_{Al} = (20\ lb_m)(453.6\ g/lb_m) = 9{,}072\ g\ Al$$

$$m'_V = (5\ lb_m)(453.6\ g/lb_m) = 2{,}268\ g\ V$$

$g \rightarrow$ moles:

$$n_{m_{Ti}} = \frac{m'_{Ti}}{A_{Ti}} = \frac{45{,}360\ g}{47.88\ g/mol} = 947.4\ mol\ Ti$$

$$n_{m_{Al}} = \frac{m'_{Al}}{A_{Al}} = \frac{9{,}072\ g}{26.98\ g/mol} = 336.2\ mol\ Al$$

$$n_{m_V} = \frac{m'_V}{A_V} = \frac{2{,}268\ g}{50.94\ g/mol} = 44.5\ mol\ V$$

now:

$$C'_{Ti} = \frac{n_{m_{Ti}}}{n_{m_{Ti}} + n_{m_{Al}} + n_{m_V}} \times 100 = \frac{947.4\ mol}{(947.4 + 336.2 + 44.5)\ mol} \times 100$$

$$= \boxed{71.3\ \text{atom\% Ti}}$$

$$C'_{Al} = \frac{n_{m_{Al}}}{n_{m_{Ti}} + n_{m_{Al}} + n_{m_V}} \times 100 = \frac{336.2 \text{ mol}}{(947.4 + 336.2 + 44.5) \text{ mol}} \times 100$$

$$= \boxed{25.3 \text{ atom \% Al}}$$

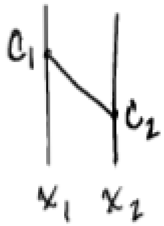
$$C'_V = \frac{n_{m_V}}{n_{m_{Ti}} + n_{m_{Al}} + n_{m_V}} = \frac{44.5 \text{ mol}}{(947.4 + 336.2 + 44.5) \text{ mol}} \times 100$$

$$= \boxed{3.4 \text{ atom \% V}}$$

totals = 100 atom % ✓

8. **(12 pts)** A sheet of steel (2.5 mm thick, cross-sectional area = 0.25 m^2) has nitrogen gas atmospheres on both sides at 1100 °C and is permitted to achieve a steady-state diffusion condition. For these conditions, the diffusion coefficient for nitrogen in steel is 1.2 x 10^{-7} m^2/s ~~and the diffusion flux is found to be 1.0 x 10^{-7} kg/m^2s~~. Also, the concentration of nitrogen in the steel at the *high*-pressure surface is 2.0 kg/m^3 and the concentration of nitrogen in the steel at the *low*-pressure surface is 0.5 kg/m^3. What is the number of kg of nitrogen that pass through the sheet per hour. Put a box around final answer.

Note: The diffusion flux should not have been provided and on exams, such a problem would not have this information provided. Thus, the answer would be determined as shown below:



$C_1 = 2.0\ kg/m^3 \qquad x_1 = 0$

$C_2 = 0.5\ kg/m^3 \qquad x_2 = 2.5 \times 10^{-3}\ m$

$$J = \frac{M}{At} \xrightarrow{so} M = JAt$$

$$J = -D\,\frac{C_2 - C_1}{x_2 - x_1}$$

$$M = -D\left(\frac{C_2 - C_1}{x_2 - x_1}\right)At$$

$$M = -(1.2 \times 10^{-7}\ m^2/s)\left(\frac{0.5 - 2\ kg/m^3}{2.5 \times 10^{-3}\ m}\right)(.25\ m^2)\left(3600\ \frac{sec}{hr}\right)$$

$$= \boxed{.0648\ kg/hr}$$

9. **(12 pts)** How many hours of carburizing are required to achieve a carbon concentration of 0.5 wt% at a position 1.5 mm into an iron-carbon alloy that initially contains 0.2 wt% C. The surface concentration is to be maintained at 0.85 wt% C and the treatment is to be conducted at 900 °C. Use the diffusion data for γ-Fe in *Table 5.2*. Consult *Table 5.1* for tabulation of error function values. **SHOW CALCULATION OF D.**

$C_0 = 0.2$ wt% C $x = .0015$ m

$C_s = 0.85$ wt% C $T = 1173$ K

$C_x = 0.5$ wt% C

① use Fick's 2nd law

$$\frac{C_x - C_0}{C_s - C_0} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$\frac{0.5 - 0.2}{0.85 - 0.2} = .462 = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$.538 = \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = \text{erf}(z)$$

② Find value of z using Table 5.1

z	erf(z)
0.50	0.5205
z	0.538
0.55	0.5633

linear interpolation →

$$\frac{z - .5}{.55 - .5} = \frac{.538 - .5205}{.5633 - .5205}$$

↓

$z = .5204$

$$z = .5204 = \frac{x}{2\sqrt{Dt}}$$

③ Must determine D [use Table 5.2] γ-Fe

$$D = D_0 \exp\left(-\frac{Q_d}{RT}\right)$$

$$= (2.3 \times 10^{-5}\ m^2/s) \exp\left(\frac{148{,}000\ J}{(8.31\ J/mol\,K)(1173\ K)}\right)$$

$$= 5.86 \times 10^{-12}\ m^2/s$$ [which matches table value]

④ solve for "t"

$$\rightarrow .5204 = \frac{x}{2\sqrt{Dt}}$$

$$\rightarrow .5204 = \frac{.0015\ m}{2\sqrt{5.86 \times 10^{-12}\ m^2/s \cdot t}}$$

$$\rightarrow .5204 = \frac{309\ s^{1/2}}{\sqrt{t}}$$

$$\rightarrow t = \left(\frac{309\ s^{1/2}}{.5204}\right)^2 = 352{,}568\ s \cdot \frac{1\ hr}{3600\ s} = \approx \boxed{98\ hr}$$